2019 BC #6
(no calculator)

(a)

The graph shown is the graph of f and a tangent line.

$f(0)=3$ from the graph.

$f'(0)=-\dfrac{3}{3/2}=-2$ from the tangent line

$f''(0)=3$ from the chart

$f^{(3)}(0)=-\dfrac{23}{2}$ from the chart

$$T_3(x)=\boxed{3-2x+\frac{3}{2!}x^2-\frac{23/2}{3!}x^3}$$

(b)

$$\boxed{e^x=1+x+\frac{x^2}{2!}+\cdots}$$

$$e^x f(x)\approx\left(1+x+\frac{x^2}{2!}+\cdots\right)\left(3-2x+\frac{3}{2!}x^2-\frac{23/2}{3!}x^3+\cdots\right)$$

$$\approx\left(3-2x+\frac{3}{2!}x^2+\cdots\right)+\left(3x-2x^2+\cdots\right)+\left(\frac{3x^2}{2!}-\cdots\right)+\cdots$$

$$\approx\boxed{3+x+\left(\frac{3}{2!}-2+\frac{3}{2!}\right)x^2}$$

(c)

$$h(1)=\int_0^1 f(t)\,dt\approx\int_0^1\left(3-2x+\frac{3}{2!}x^2-\frac{23/2}{3!}x^3\right)dx$$

$$=\left[3x-x^2+\frac{1}{2}x^3-\frac{23}{48}x^4\right]_0^1$$

$$=\boxed{3-1+\frac{1}{2}-\frac{23}{48}}\quad\frac{97}{48}$$

(d)

The alternating series error bound is the first unused term of $h(x)$ found in part (c).

The first unused term of $f(x)$ from part (a) is $\dfrac{54}{4!}x^4$ so the

first unused term of the approximation in part (c) $h(x)$ is

$$\int_0^1\frac{54}{4!}x^4\,dx=\frac{54}{4!\cdot 5}x^5\Big]_0^1$$

$$=\frac{54}{120}$$

$$=\frac{9}{20}$$

$$=0.45$$

This is the alternating series error bound so the approximation found in part (c) differs from $h(1)$ by at most 0.45.