2019 BC #2
(calculator-active)

(a) $r(\theta) = 3\sqrt{\theta}\sin\left(\theta^2\right)$ for $0 \le \theta \le \sqrt{\pi}$ Area $= \dfrac{1}{2}\displaystyle\int_0^{\sqrt{\pi}} \left(r(\theta)\right)^2 d\theta \approx \boxed{3.534291735}$
(b) The distance from the origin to a point on the curve is r. $r_{avg} = \dfrac{1}{\sqrt{\pi}-0}\displaystyle\int_0^{\sqrt{\pi}} r(\theta)d\theta \approx \boxed{1.57993277}$
(c) Since the slope of the line is m, then $m = \tan\theta$. So $\theta = \tan^{-1} m$ is where the line intersects $r(\theta)$. So, the area from 0 to $\tan^{-1} m$ is equal to the area from $\tan^{-1} m$ to $\sqrt{\pi}$: $\boxed{\dfrac{1}{2}\displaystyle\int_0^{\tan^{-1} m} \left(r(\theta)\right)^2 d\theta = \dfrac{1}{2}\displaystyle\int_{\tan^{-1} m}^{\sqrt{\pi}} \left(r(\theta)\right)^2 d\theta}$
(d) As $k \to \infty$, $r = k\cos\theta$ is a circle that gets bigger and $\theta \to \dfrac{\pi}{2}$. So, $\displaystyle\lim_{k\to\infty} A(k) = \dfrac{1}{2}\int_0^{\pi/2} \left(r(\theta)\right)^2 d\theta \approx \boxed{3.324470722}$