

2019 RELEASED FREE RESPONSE SOLUTIONS – MR. CALCULUS

**2019 AB/BC #4
(no calculator)**

(a)

Since the radius of the barrel does not change, then $\frac{dr}{dt} = 0$ and I can substitute 1 for r right away.

$$\text{So } V = \pi(1^2)h = \pi h$$

$$\frac{dV}{dt} = \pi \frac{dh}{dt} = \pi \left(-\frac{1}{10} \sqrt{h} \right)$$

$$\left. \frac{dV}{dt} \right|_{h=4} = \left[\pi \left(-\frac{1}{10} \sqrt{h} \right) \frac{ft^3}{sec} \right]_{h=4} = -\frac{\pi}{5} \frac{ft^3}{sec}$$

(b)

$$\frac{dh}{dt} = -\frac{1}{10} \sqrt{h} = -\frac{1}{10} h^{1/2}$$

$$\frac{d}{dt} \left(\frac{dh}{dt} \right) = \frac{d^2h}{dt^2} = -\frac{1}{20} h^{-1/2} \left(\frac{dh}{dt} \right) = -\frac{1}{20} h^{-1/2} \left(-\frac{1}{10} h^{1/2} \right) = \frac{1}{200}$$

$$\left. \frac{d^2h}{dt^2} \right|_{h=3} = \frac{1}{200} > 0$$

So the rate of change of the height of the water when $h = 3$ is increasing.

(c)

$$\frac{dh}{dt} = -\frac{1}{10} h^{1/2}$$

$$\frac{dh}{\sqrt{h}} = -\frac{1}{10} dt$$

$$\int h^{-1/2} dh = \int -\frac{1}{10} dt$$

$$2h^{1/2} = -\frac{1}{10}t + C \quad \text{Since when } t = 0, h = 5: 2\sqrt{5} = C$$

$$2\sqrt{h} = -\frac{1}{10}t + 2\sqrt{5}$$

$$\sqrt{h} = -\frac{1}{20}t + \sqrt{5}$$

$$h = \left(-\frac{1}{20}t + \sqrt{5} \right)^2$$