

**2019 AB/BC #3  
(no calculator)**

(a)

$$\int_{-6}^5 f(x) dx = \int_{-6}^{-2} f(x) dx + \int_{-2}^5 f(x) dx$$

$$\text{So, } \int_{-6}^{-2} f(x) dx = 7 - \int_{-2}^5 f(x) dx = \boxed{7 - \left[ \frac{1}{2}(1)(1) - \frac{1}{2}\left(\frac{3}{2}\right)(1) + \frac{1}{2}\left(\frac{3}{2}\right)(3) + \left(3^2 - \frac{1}{4}\pi(3^2)\right) \right]}$$

(b)

$$\begin{aligned} \int_3^5 (2f'(t) + 4) dt &= 2 \int_3^5 f'(t) dt + \int_3^5 4 dt \\ &= 2[f(5) - f(3)] + 8 \\ &= \boxed{2[0 - (3 - \sqrt{5})] + 8} \quad 2 + 2\sqrt{5} \end{aligned}$$

(c)

The absolute maximum value of  $g$  on  $-2 \leq x \leq 5$  will occur at a critical point of  $g$  on the interval or will be  $g(-2)$  or  $g(5)$ .

$$g'(x) = f(x)$$

$$g'(x) = f(x) = 0 \Rightarrow x = -1, \frac{1}{2}, 5$$

$$g(-2) = \int_{-2}^{-2} f(x) dx = 0$$

$$g(-1) = \int_{-2}^{-1} f(x) dx = \frac{1}{2}$$

$$g\left(\frac{1}{2}\right) = \int_{-2}^{1/2} f(x) dx = \frac{1}{2} - \frac{3}{4} = -\frac{1}{4}$$

$$g(5) = \int_{-2}^5 f(x) dx = \frac{1}{2}(1)(1) - \frac{1}{2}\left(\frac{3}{2}\right)(1) + \frac{1}{2}\left(\frac{3}{2}\right)(3) + \left(3^2 - \frac{1}{4}\pi(3^2)\right) = 11 - \frac{9\pi}{4} \text{ (see part (a))}$$

$$\text{The absolute maximum value of } g \text{ on } -2 \leq x \leq 5 \text{ is } \boxed{11 - \frac{9\pi}{4}}$$

(d)

Note: We can see that  $f$  is both continuous and differentiable at  $x = 1$ .

$$\lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} = \frac{10^1 - 3f'(1)}{f(1) - \arctan 1} = \frac{\boxed{10 - 3\left(\frac{4}{2}\right)}}{\boxed{1 - \frac{\pi}{4}}}$$