2019 RELEASED FREE RESPONSE SOLUTIONS – MR. CALCULUS

2019 AB/BC #1
(calculator-active)

(a)
Since $E(t)$ is the rate at which fish enter the lake, then the number of fish

that enter the lake from $0 \le t \le 5$ is $\int_0^5 E(t)\,dt \approx 153.4576901$ $\boxed{153\,fish}$

(b)
Since $L(t)$ is the rate that fish leave the lake, then the average number of fish

that leave the lake from $0 \le t \le 5$ is $\frac{1}{5}\int_0^5 L(t)\,dt \approx \boxed{6.059037771\frac{fish}{hour}}$ $6.059\frac{fish}{hour}$

(c)

Let the number of fish in the lake at any time $t \ge 0$ be $F(t) = F(0) + \int_0^t \big(E(t) - L(t)\big)\,dt.$

The greatest number of fish in the lake when $0 \le t \le 8$ will occur when $F'(t) = 0$ on that interval or at $F(0)$ or at $F(8)$. But, we were not given $F(0)$, the number of fish in the lake when $t = 0$.

$F'(t) = E(t) - F(t) = 0$ only when $t \approx 6.203564 = a$ for $0 \le t \le 8$.

Since $E(t) > L(t)$ for $0 \le t < a$ then $F'(t) > 0$ for $0 \le t < a$ and

since $E(t) < L(t)$ for $a < t \le 8$ then $F'(t) < 0$ for $a < t \le 8$.

So $F(a)$ is a relative maximum but since it is the only critical point on the interval, it is absolute maximum as well. Hence the greatest number of fish in the lake for $0 \le t \le 8$ occurs when $\boxed{t = a\ hours}$.

(d)
$F(t)$ is the number of fish in the lake at any time, t.

$F'(t) = E(t) - L(t)$ is the rate of change in the number of fish in the lake at any time, t.

$F''(t) = E'(t) - L'(t)$

$F''(5) = E'(5) - L'(5) \approx -6.802 - 3.921 < 0$

Hence the rate of change in the number of fish in the lake

is $\boxed{\text{decreasing}}$ at $t = 5$.